Reports of the Department of Geodetic Science
Report No. 189

QUADRATURE ERRORS IN THE PARTIAL DERIVATIVES REQUIRED FOR THE DIRECT RECOVERY OF GRAVITY ANOMALIES FROM SATELLITE OBSERVATIONS

by
D. P. Hajela

Prepared for
National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

Grant No. NGR 36-008-161
OSURF Project No. 3210

The Ohio State University
Research Foundation
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Abstract

The equations of motion of a geodetic satellite in the earth's gravitational field expressed by gravity anomalies require the evaluation, amongst others, of the partial derivatives of the disturbing force with respect to individual gravity anomalies. These derivatives would be in error if evaluated using coordinates at the center points of the mean gravity anomaly blocks.

This report discusses how these blocks should be subdivided so that the partial derivatives could be numerically evaluated for each subdivision, and then finally meaned to give the value representative of the whole blocks, with accuracies better than 2 - 3% for all blocks. The number of subdivisions is large for the blocks nearest to the satellite subpoint and decreases away from it. The actual values of this spherical distance and the actual subdivision of the mean gravity anomaly blocks has been determined numerically for 184 $15^\circ \times 15^\circ$ equal area blocks. Satellite heights above the earth of 400 km, 800 km and 1600 km have been considered. The computer times for the suggested scheme have been compared with alternative solutions.

Foreword

This report was prepared by Mr. D. P. Hajela, Graduate Research Associate, Department of Geodetic Science, under NASA Grant NGR 36-008-161, The Ohio State University Research Foundation Project No. 3210 which is under the direction of Professor Richard H. Rapp. The contract covering this research is administered through the Goddard Space Flight Center, Mr. L. H. Carpenter, Technical Officer.

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1. Introduction

The predominant force acting on artificial earth satellites used for geodetic purposes is that of the earth's gravity field. If we choose to represent it by a set of mean gravity anomalies over specified blocks and referred to a defined reference surface, we need to evaluate the effect of each gravity anomaly at the given satellite position. As this has to be repeated for each satellite position being considered, for example in the numerical integration approach of orbital and trajectory analysis, any simplification in the practical evaluation consistent with the required accuracy would be significant.

The following specific problems have been examined in this paper:

(a) Is it adequate to consider the effect from the center of the block over which the mean gravity anomaly is given, or should the effect be meaned over several points in the block? If so, how should these points be chosen?

(b) Is it adequate to have the computations being meaned over points in the block for only a few blocks near the satellite subpoint; and if so, up to what distance from the satellite subpoint?

(c) Will it be possible to ignore the effect of some blocks altogether, which are far removed from the satellite subpoint?

(d) How will the above conclusions vary with change in the height of the satellite?

2. The Basic Equations

Following Rapp (1971), the equations of motion of the satellite in an inertial coordinate system (x, y, z) at time t, measured from an initial epoch t_o, in terms of acceleration components $\ddot{x}, \ddot{y}, \ddot{z}$, may be expressed in a general form as:

$$
\begin{aligned}
\ddot{x} &= f(t, x, y, z) = f(t, x_o, y_o, z_o, \dot{x}_o, \dot{y}_o, \dot{z}_o, \Delta g_1, \Delta g_2, \ldots \Delta g_n) \\
\ddot{y} &= g(t, x, y, z) = g(t, x_o, y_o, z_o, \dot{x}_o, \dot{y}_o, \dot{z}_o, \Delta g_1, \Delta g_2, \ldots \Delta g_n) \qquad (1) \\
\ddot{z} &= h(t, x, y, z) = h(t, x_o, y_o, z_o, \dot{x}_o, \dot{y}_o, \dot{z}_o, \Delta g_1, \Delta g_2, \ldots \Delta g_n)
\end{aligned}
$$

where x_0, y_0, z_0 and $\dot{x}_0, \dot{y}_0, \dot{z}_0$ are the initial position and velocity components of the satellite at epoch t_0, and $\Delta g_1, \Delta g_2, \ldots \Delta g_n$ are the mean gravity anomalies.

Using β_k for any one of the individual gravity anomalies or the initial position and velocity components $(x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0)$, the variational equations with respect to β_k may be expressed as:

$$\frac{\partial \ddot{x}}{\partial \beta_k} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial \beta_k} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial \beta_k} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial \beta_k} + \frac{\partial f}{\partial \beta_k}$$

$$\frac{\partial \ddot{y}}{\partial \beta_k} = \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial \beta_k} + \frac{\partial g}{\partial y} \cdot \frac{\partial y}{\partial \beta_k} + \frac{\partial g}{\partial z} \cdot \frac{\partial z}{\partial \beta_k} + \frac{\partial g}{\partial \beta_k} \tag{2}$$

$$\frac{\partial \ddot{z}}{\partial \beta_k} = \frac{\partial h}{\partial x} \cdot \frac{\partial x}{\partial \beta_k} + \frac{\partial h}{\partial y} \cdot \frac{\partial y}{\partial \beta_k} + \frac{\partial h}{\partial z} \cdot \frac{\partial z}{\partial \beta_k} + \frac{\partial h}{\partial \beta_k}$$

We will confine our discussion to the evaluation of partial derivatives with respect to the gravity anomalies and use the notation:

$$C_x = \frac{\partial f}{\partial \beta_k}, \quad C_y = \frac{\partial g}{\partial \beta_k}, \quad C_z = \frac{\partial h}{\partial \beta_k} \tag{3}$$

where the evaluation of C_x, C_y, C_z will be done for each individual gravity anomaly $\Delta g_1, \Delta g_2, \ldots \Delta g_n$.

3. Equations for C_x, C_y, C_z

Considering the equations of motion (1) of the satellite only due to the earth's gravity potential composed of the normal part U and the disturbing part T, we may write:

$$\ddot{x} = \frac{\partial V}{\partial x} = \frac{\partial U}{\partial x} + \frac{\partial T}{\partial x} \tag{4}$$

and similar expressions for $\ddot{y}$, $\ddot{z}$, which when combined with equations (3) and

(2) give:

$$C_{x_i} = \frac{\partial}{\partial \Delta g_i} \left(\frac{\partial T}{\partial x} \right) \tag{5}$$

and similar expressions for C_{y_i}, C_{z_i} for an individual gravity anomaly Δg_i.

As the partial derivatives of the disturbing potential with respect to the satellite position in the x, y, z coordinate system may be expressed with respect to r, ψ, and λ, being respectively the geocentric radius vector, latitude and longitude of the satellite; e.g.,

$$\frac{\partial T}{\partial x} = \frac{\partial r}{\partial x} \cdot \frac{\partial T}{\partial r} + \frac{\partial \psi}{\partial x} \cdot \frac{\partial T}{\partial \psi} + \frac{\partial \lambda}{\partial x} \cdot \frac{\partial T}{\partial \lambda} \tag{6}$$

and as the value of $\frac{\partial T}{\partial r}$, $\frac{\partial T}{\partial \psi}$, $\frac{\partial T}{\partial \lambda}$ may be taken from literature (eg. Heiskanen and Moritz, 1967) in terms of the generalized Stokes' function, we may express the value of C_x, C_y, C_z following Rapp (1971) as:

$$\begin{bmatrix} C_{x_i} \\ C_{y_i} \\ C_{z_i} \end{bmatrix} = \begin{bmatrix} \frac{\partial r}{\partial x} & \frac{\partial \alpha}{\partial x} & \frac{\partial \psi}{\partial x} \\ \frac{\partial r}{\partial y} & \frac{\partial \alpha}{\partial y} & \frac{\partial \psi}{\partial y} \\ \frac{\partial r}{\partial z} & \frac{\partial \alpha}{\partial z} & \frac{\partial \psi}{\partial z} \end{bmatrix} \begin{bmatrix} A_i \\ C_i \\ B_i \end{bmatrix} \tag{7}$$

where $\frac{\partial \alpha}{\partial x}$, $\frac{\partial \alpha}{\partial y}$, $\frac{\partial \alpha}{\partial z}$ have been used in place of $\frac{\partial \lambda}{\partial x}$, $\frac{\partial \lambda}{\partial y}$, $\frac{\partial \lambda}{\partial z}$ these being equal at any given epoch as α, the Right Ascension of the satellite is given in terms of λ and θ, the Greenwich Siderial time by

$$\alpha = \lambda + \theta \tag{8}$$

The values of the coefficients A_i, B_i, C_i corresponding to a given gravity anomaly Δg_i may be expressed as:

$$A_i = \frac{R}{4\Pi} \frac{\partial S(r,\psi^*)}{\partial r} d\sigma$$

$$B_i = \frac{-R}{4\Pi} \frac{\partial S(r,\psi^*)}{\partial \psi^*} \cos\alpha^* d\sigma \qquad (9)$$

$$C_i = \frac{-R\cos\psi}{4\Pi} \frac{\partial S(r,\psi^*)}{\partial \psi^*} \sin\alpha^* d\sigma$$

where R is the radius of a sphere approximating the earth, $d\sigma$ is the area of the gravity anomaly block/subblock ψ^* is the spherical distance and α^* is the azimuth from the satellite subpoint to the gravity anomaly block/subblock and $S(r,\psi^*)$ is the generalized Stokes' function. Its value and that of its partial derivatives with respect to r and ψ^* is given as in Heiskanen and Moritz (1967) by:

$$S(r,\psi^*) = t\left[\frac{2}{D} + 1 - 3D - t\cos\psi^*\left(5 + 3\ln\frac{1-t\cos\psi^* + D}{2}\right)\right]$$

$$\frac{\partial S}{\partial r} = \frac{-t^2}{R}\left[\frac{1-t^2}{D^3} + \frac{4}{D} + 1 - 6D - t\cos\psi^*\left(13 + 6\ln\frac{1-t\cos\psi^* + D}{2}\right)\right] \qquad (10)$$

$$\frac{\partial S}{\partial \psi^*} = -t^2\sin\psi^*\left[\frac{2}{D^3} + \frac{6}{D} - 8 - 3\frac{1-t\cos\psi^* - D}{D\sin^2\psi^*} - 3\ln\frac{1-t\cos\psi^* + D}{2}\right]$$

where $t = \frac{R}{r}$ and $D = (1-2t\cos\psi^* + t^2)^{\frac{1}{2}}$.

The partial derivatives of r, α, ψ with respect to the satellite position are obtained from:

$$x = r\cos\psi\cos\alpha$$

$$y = r\cos\psi\sin\alpha$$

$$z = r\sin\psi \qquad (11)$$

$$r^2 = x^2 + y^2 + z^2$$

Finally, the values of ψ^* and α^* may be obtained in terms of the geocentric latitude ψ and longitude λ of the satellite subpoint and the corresponding values ψ_g and λ_g of the gravity anomaly block/sub block by:

$$
\begin{aligned}
\cos \psi^* &= \sin \psi \sin \psi_g + \cos \psi \cos \psi_g \cos(\lambda_g - \lambda) \\
\sin \alpha^* &= \frac{\cos \psi_g \sin(\lambda_g - \lambda)}{\sin \psi^*} \qquad\qquad (12) \\
\cos \alpha^* &= \frac{\cos \psi \sin \psi_g - \sin \psi \cos \psi_g \cos(\lambda_g - \lambda)}{\sin \psi^*}
\end{aligned}
$$

The value of C_{x_i}, C_{y_i}, C_{z_i} for each of the gravity anomalies Δg_i ($\Delta g_1, \Delta g_2, \ldots \Delta g_n$) for a given satellite position (x, y, z) at epoch t is thus given by substituting equations (9) to (12) in equation (7).

We have confined our discussion in this paper to the evaluation of C_x, C_y, C_z numerically. The purpose of the investigation has been to compute them accurately enough in an optimum manner, thereby saving computer time, by omitting excessive computations, which do not lead to appreciable increase in accuracy. The computations were done on an IBM System/370 Model 165 computer.

4. Test Data

The gravity anomalies used to define the earth's gravity field were 184 $15^\circ \times 15^\circ$ equal area mean anomalies, as listed by Obenson [1970, pp. 127 - 128] and used by Haverland [1971].

The satellite orbit was generated using a slightly modified version of a Cowell orbit generation program using an eleventh order Cowell integration with a fixed step size of 60 seconds. The earth's gravitational field alone was considered using potential coefficients $\overline{C}_{2,0}$ and $\overline{C}_{4,0}$, as in Rapp (1971).

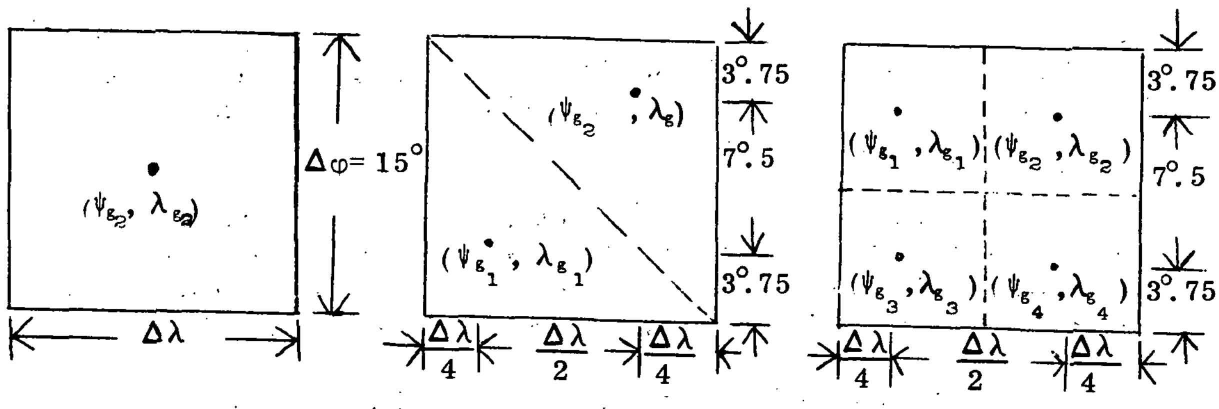
The computer program as developed by Haverland (1971) and subsequently revised by Rapp (unpublished) was used. A few modifications were made but the orbital parameters were retained, except for different values for the semi-major axis to give approximate satellite heights above the earth of 400 km, 800 km, and 1600 km to cover the usual range of geodetic satellites. For still higher satellites, the conclusions drawn in this paper will be on the safe side.

The other orbital parameters were:

eccentricity	$e = 0.07061...$
inclination	$i = 59^\circ.386...$
argument of perigee	$\omega = 312^\circ.74...$
longitude of ascending node	$\Omega = 263^\circ.74...$
mean anomaly at epoch	$M_o = 0.23176...$ revolutions

5. Block Subdivisions

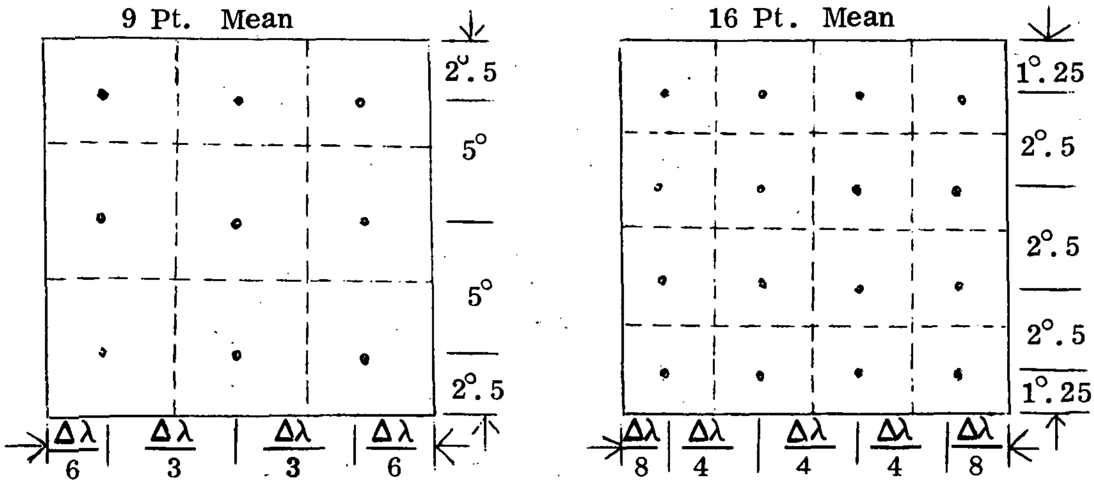
The most direct evaluation of C_x, C_y, C_z would take place by evaluating equation (9) using coordinates ofthe center of the 15° block only. Such an evaluation may suffer from a numerical integration error. Consequently, we first examine a subdivision of the block considering 2 sub blocks and 4 sub blocks respectively. The value of C_x, C_y, C_z representative of the whole block would be obtained as a mean of the sub block values. These means could then be termed as 2 point mean and 4 point mean respectively, and could be used in place of one point (center point) value of the whole $15^\circ \times 15^\circ$ block. As the latitudinal extent, $\Delta\varphi$ of the blocks was uniformly 15°, but as the longitudinal extent $\Delta\lambda$, increased towards the poles up to 120° from the 15° value at the equator to keep the blocks nominally as equal area, the scheme of subdivision of the blocks into sub blocks was as below:



The value of C_x, C_y, C_z was computed based on a 1 point (center) evaluation and as 2 point mean and 4 point mean values for all the 184 $15^\circ \times 15^\circ$ blocks. The differences (2 point mean - 1 point value) and (4 point mean - 1 point value) were computed individually for each mean gravity anomaly block. A root mean square value was then obtained for these differences over all the 184 blocks and compared against the root mean square value over the 184 blocks of the 2 point mean and 4 point mean values of C_x, C_y, C_z, and expressed as a percentage.

The results are given in Table 1 for four different satellite heights. It was found that the 2 point mean value, when compared with the 4 point mean value, was no better than the one point value, but worse in most cases. This is perhaps due to the assymetric position of the centers of the sub blocks in the case of 2 point means, as compared to the center of the whole block. Accordingly, 2 point means have not been considered further. For the same reasons, no other assymetric subdivisions of the block was considered.

Following these first results, further symmetric subdivisions were chosen to obtain more refined values. Specifically a 9 point and a 16 point mean was evaluated. This subdivision is shown below:



The results in Table 1 show the effect of increasing the number of sub blocks on the values of C_x, C_y, C_z. It is reasonable to expect that the 16 point means are more accurate that 9 point means, which are more accurate than 4 point means, etc. However, as root mean square values over all the 184 blocks have been considered in Table 1, the root mean square value of differences (16 point mean - 1 point value), (16 point mean - 4 point value), etc. come out to be rather large. As we see later, these differences are predominantly large for individual gravity anomaly blocks nearest to the satellite subpoint, and are much smaller away from it.

The results in Table 1 may be used as a guide in estimating the error in computing C_x, C_y, C_z when using a 1, 4, or 9 point mean if we accept the 16 point mean as being correct.

TABLE 1

Effect of Number of Block Subdivisions on Accuracy of Computation

	1 Pt. Value Compared with 2,4,9,16 Pt. Means												2 Pt. Mean Compared with		4 Pt. Mean Compared with		Remarks
	Root Mean Square Difference				Root Mean Square Difference				% Difference = RMS diff./RMS Value x100				4 Pt. Mean	16 Pt. Mean	9 Pt. Mean	16 Pt. Mean	
	2 Pt	4 Pt	9 Pt	16 Pt	2 Pt	4 Pt	9 Pt	16 Pt	2 Pt	4 Pt	9 Pt	16 Pt	% Difference = RMS diff./RMS Value x 100				
C_x	.0092	.0105	.0113	.0115	.0272	.0257	.0252	.0251	33.8	40.9	45.0	45.9	10.1	13.0	3.3	4.1	r≃7187 km
C_y	.0078	.0015	.0009	.0009	.0320	.0299	.0294	.0293	24.4	5.1	3.1	3.1	22.6	26.7	3.6	4.1	h≃816 km
C_z	.0110	.0100	.0107	.0110	.0306	.0307	.0303	.0302	36.1	32.6	35.3	36.3	6.2	5.3	2.4	3.2	t = 1.0min
C_x	.0025	.0033	.0043	.0046	.0274	.0256	.0249	.0248	9.3	12.9	17.2	18.4	11.5	16.2	4.0	5.2	r≃7219 km
C_y	.0095	.0043	.0043	.0043	.0261	.0276	.0274	.0274	36.6	15.7	15.8	15.8	25.1	24.1	1.0	1.4	h≃ 848 km
C_z	.0024	.0025	.0034	.0037	.0319	.0304	.0298	.0296	7.4	8.2	11.5	12.4	10.9	14.6	3.3	4.2	t = 2.0min
C_x	.0038	.0027	.0034	.0036	.0249	.0249	.0244	.0242	15.3	10.9	14.0	15.0	11.6	12.4	3.2	4.2	r≃7250 km
C_y	.0057	.0023	.0025	.0025	.0259	.0269	.0267	.0266	22.1	8.5	9.2	9.5	21.4	20.1	1.9	2.6	h≃ 879 km
C_z	.0052	.0028	.0033	.0034	.0283	.0290	.0287	.0285	18.2	9.7	11.4	11.9	17.1	17.0	2.3	3.0	t = 3.0min
C_x	.0046	.0024	.0031	.0033	.0231	.0244	.0238	.0237	19.7	9.7	12.9	14.0	14.3	12.6	3.4	4.5	r≃7280 km
C_y	.0032	.0015	.0020	.0022	.0282	.0265	.0261	.0260	11.7	5.5	7.7	8.5	14.6	17.2	2.4	3.2	h≃909 km
C_z	.0068	.0035	.0037	.0037	.0272	.0277	.0276	.0276	24.9	12.5	13.3	13.5	20.8	20.3	1.2	1.6	t=4.0min

6. Number of Blocks for Computation of 16 Point/ 9 Point Means

The predominant difference between the 16, 9 and 4 point means occur for those mean gravity anomaly blocks, which are nearest to the satellite subpoint, within a spherical distance ψ^* (equation (12)) of 10° - 20°.

Tables 2, 3 and 4 show respectively for satellite heights above earth of about 400, 800 and 1600 km, the values of C_x, C_y, C_z for 16, 9, 4 point means and center (1 point) value of all mean gravity anomaly blocks within a spherical distance ψ^* of 20° from the satellite subpoint.

It is clear from these results that center point value cannot be used for $\psi^* < 20^\circ$, and even 4 point mean is in considerable error for $\psi < 15^\circ$ or so. For the first nearest block with $\psi^* < 5^\circ$, even the 9 point mean appears to be in error by 1% to 4%.

TABLE 2

C_x, C_y, C_z Values for Nearest Blocks $\psi^* < 20°$ Satellite Ht. ≈ 400 km

Block No.	Sph. Dist. ψ^* Deg.	16Pt. Mean	1Pt. Value	Difference 16Pt - 1Pt	Percentage $\frac{(16p-1p)100}{16p}$	9Pt. Mean	Difference 16Pt - 9Pt	Percentage $\frac{(16p-9p)100}{16p}$	4Pt. Mean	Difference 16Pt - 4Pt	Percentage $\frac{(16p-4p)100}{16p}$	Remarks
C_x	Values											$r \simeq 6779$km
139	4.6	-.3649	-.8302	.4653	127.5	-.3499	-.0150	4.1	-.4750	.1101	30.2	$h \simeq 408$ km
140	11.1	.2159	.2304	-.0145	6.7	.2171	-.0012	0.6	.2197	-.0038	1.8	$t = 1.0$ min
158	12.7	-.1291	-.1236	-.0054	4.2	-.1281	-.0010	0.7	-.1241	-.0050	3.9	
117	18.2	-.0274	-.0269	-.0005	1.9	-.0273	-.0001	0.4	-.0270	-.0004	1.5	
157	18.8	-.1477	-.1439	-.0037	2.5	-.1474	-.0003	0.2	-.1467	-.0010	0.7	
118	19.4	.0733	.0731	.0002	0.3	.0732	.0001	0.1	.0732	.0001	0.1	
159	19.4	.0425	.0397	.0028	6.5	.0424	.0001	0.2	.0421	.0004	0.9	
C_y	Values											Discordant Value
139	4.6	.3232	.0189	.3042	94.1	.3213	.0019	0.6	.5187			
140	11.1	.2410	.2090	.0320	13.3	.2386	.0024	1.0	.2352	.0058	2.4	
158	12.7	.2382	.2147	.0235	9.9	.2360	.0022	0.9	.2309	.0073	3.1	
117	18.2	-.0554	-.0571	.0016	2.9	-.0556	.0002	0.4	-.0558	.0004	0.7	
157	18.8	-.0130	-.0148	.0018	14.1	-.0131	.0001	0.6	-.0133	.0003	2.3	
118	19.4	-.0047	-.0059	.0012	26.6	-.0048	.0001	2.1	-.0051	.0004	8.5	
159	19.4	.1343	.1252	.0090	6.7	.1337	.0006	0.4	.1322	.0021	1.6	
C_z	Values											
139	4.6	-.5217	-.8512	.3295	63.2	-.5172	-.0045	0.9	-.5345	.0128	2.5	
140	11.1	-.1576	-.1371	-.0206	13.1	-.1545	-.0031	2.0	-.1459	-.0117	7.4	
158	12.7	.0911	.1015	-.0104	11.4	.0921	-.0010	1.1	.0939	-.0028	3.1	
117	18.2	-.1511	-.1447	-.0064	4.2	-.1507	-.0004	0.3	-.1495	-.0016	1.1	
157	18.8	-.0381	-.0350	-.0031	8.1	-.0379	-.0002	0.5	-.0372	-.0009	2.4	
118	19.4	-.1280	-.1224	-.0056	4.4	-.1277	-.0003	0.2	-.1268	-.0012	0.9	
159	19.4	.0393	.0406	-.0013	3.2	.0394	-.0001	0.2	.0397	-.0004	1.0	

TABLE 3

C_x, C_y, C_z Values for Nearest Blocks $\psi^* < 20^\circ$ Satellite Ht. $\approx$ 800 km.

Block No.	Sph. Dist. ψ^* Deg.	16Pt. Mean	1Pt. Mean	Difference 16Pt - 1Pt	Percentage $\frac{(16p-1p)100}{16p}$	9Pt. Mean	Difference 16Pt- 9 Pt	Percentage $\frac{(16p-9p)100}{16p}$	4Pt. Mean	Difference 16Pt-4Pt	Percentage $\frac{(16p-4p)100}{16p}$	Remarks
C_x	Values											$r \simeq 7187$ km
139	4.3	-.1936	-.3478	.1542	79.6	-.1958	.0022	1.1	-.2062	.0126	6.5	$h \simeq 816$ km
140	11.3	.1050	.1298	-.0249	23.7	.1066	-.0016	1.5	.1108	-.0058	5.5	t = 1.0 min
158	13.0	-.0917	-.0915	-.0002	0.2	-.0916	-.0001	0.1	-.0908	-.0009	1.0	
117	17.9	-.0266	-.0258	-.0008	3.1	-.0265	-.0001	0.4	-.0262	-.0004	1.5	
157	18.6	-.1182	-.1175	-.0007	0.6	-.1182	.0000	0.0	-.1181	-.0001	0.1	
118	19.3	.0519	.0537	-.0018	3.4	.0520	-.0001	0.2	.0523	-.0004	0.8	
159	19.7	.0268	.0266	.0002	0.8	.0269	-.0001	0.4	.0270	-.0002	0.7	
C_y	Values											
139	4.3	.1985	.1967	.0018	0.9	.2006	-.0021	1.1	.2148	-.0163	8.2	
140	11.3	.1774	.1732	.0043	2.4	.1775	-.0001	0.1	.1778	-.0004	0.2	
158	13.0	.1789	.1723	.0066	3.7	.1785	.0004	0.2	.1774	.0015	0.8	
117	17.9	-.0264	-.0309	.0045	17.1	-.0267	.0003	1.1	-.0275	.0011	4.2	
157	18.6	.0043	.0011	.0032	74.4	.0041	.0002	4.6	.0036	.0007	16.3	
118	19.3	.0097	.0070	.0027	28.0	.0095	.0002	2.1	.0090	.0007	7.2	
159	19.7	.1086	.1033	.0053	4.9	.1084	.0002	0.2	.1076	.0010	0.9	
C_z	Values											
139	4.3	-.2867	-.4341	.1474	51.4	-.2899	-.0032	1.1	-.2988	.0121	4.2	
140	11.3	-.1248	-.1168	-.0080	6.4	-.1242	-.0006	0.5	-.1220	-.0028	2.2	
158	13.0	.0291	.0452	-.0161	55.3	.0303	-.0012	4.1	.0333	-.0042	14.4	
117	17.9	-.1279	-.1249	-.0030	2.4	-.1277	-.0002	0.2	-.1272	-.0007	0.5	
157	18.6	-.0399	-.0366	-.0032	8.1	-.0397	-.0002	0.5	-.0390	-.0009	2.2	
118	19.3	-.1072	-.1040	-.0032	2.9	-.1070	-.0002	0.2	-.1066	-.0006	0.5	
159	19.7	.0201	.0234	-.0033	16.3	.0204	-.0003	1.4	.0209	-.0008	4.0	

TABLE 4

C_x, C_y, C_z Values for Nearest Blocks $\psi^* < 20^\circ$ Satellite Ht. $\approx$1600 km

Block No.	Sph. Dist. ψ^* Deg.	16Pt. Mean	1Pt. Mean	Difference 16Pt - 1Pt	Percentage $\frac{(16p-1p)100}{16p}$	9Pt. Mean	Difference 16Pt - 9 Pt	Percentage $\frac{(16p-9p)100}{16p}$	4Pt. Mean	Difference 16Pt-4Pt	Percentage $\frac{(16p-4p)100}{16p}$	Remarks
C_x	Values											$r \simeq 7965$km
139	3.8	-.0720	-.0950	.0230	31.9	-.0727	.0007	0.9	-.0747	.0027	3.8	$h \simeq 1594$km
140	11.6	.0319	.0415	- .0096	30.3	.0323	-.0004	1.4	.0336	-.0018	5.5	t = 1.0 min
158	13.3	-.0457	-.0477	.0020	4.4	-.0458	.0001	0.2	-.0461	.0004	0.9	
117	17.4	-.0199	-.0198	- .0001	0.4	-.0199	.0000	0.0	-.0199	.0000	0.1	
157	18.3	-.0707	-.0730	.0023	3.3	-.0708	.0001	0.2	-.0712	.0005	0.8	
118	19.1	.0252	.0275	- .0023	9.2	.0254	-.0001	0.5	.0257	-.0005	1.9	
159	20.2	.0110	.0118	- .0008	7.3	.0111	-.0001	0.5	.0112	-.0002	2.1	
C_y	Values											
139	3.8	.0961	.1104	-.0144	14.9	.0967	-.0006	0.6	.0986	-.0002	2.6	
140	11.6	.0937	.0992	-.0055	5.8	.0940	-.0003	0.4	.0950	-.0012	1.3	
158	13.3	.0963	.0992	-.0029	3.0	.0965	-.0002	0.2	.0970	-.0007	0.7	
117	17.4	.0008	-.0027	.0035	430.2	.0006	.0002	22.2	.0001	.0007	86.4	
157	18.3	.0163	.0144	.0019	11.9	.0162	.0001	0.6	.0159	.0004	2.4	
118	19.1	.0182	.0165	.0017	9.4	.0181	.0001	0.5	.0178	.0003	1.9	
159	20.2	.0682	.0675	.0007	1.0	.0682	.0000	0.0	.0682	.0000	0.1	
C_z	Values											
139	3.8	-.1149	-.1417	.0268	23.3	-.1158	.0009	0.9	-.1185	.0035	3.1	
140	11.6	-.0674	-.0687	.0013	1.9	-.0675	.0001	0.1	-.0677	.0034	0.5	
158	13.3	-.0044	.0025	-.0069	156.0	-.0041	-.0003	7.7	-.0030	-.0013	30.8	
117	17.4	-.0818	-.0835	.0016	2.0	-.0820	.0001	0.1	-.0822	.0004	0.5	
157	18.3	-.0325	-.0312	-.0013	4.1	-.0324	-.0001	0.2	-.0322	-.0003	0.9	
118	19.1	-.0691	-.0696	.0005	0.7	-.0692	.0001	0.1	-.0693	.0002	0.3	
159	20.2	.0036	.0062	-.0026	73.9	.0037	-.0001	3.9	.0041	-.0005	15.2	

We may therefore conclude that for errors due to subdivision of blocks not to exceed about 2%, a 16 point mean should be taken when the center (φ_g, λ_g) of mean gravity anomaly block is less than 10° from the satellite subpoint, and a 9 point mean when $10^\circ < \psi^* < 15^\circ$. For determining the value of ψ^* up to which 4 point mean should be taken, we should examine further results and this has been done in Section 7.

We also find from Tables 2, 3, 4 that values of C_x, C_y, C_z decrease with height of the satellite and the percentage value of difference of means (eg. 16 pt. mean - 4 pt. mean) also becomes less. The limits of $\psi^* < 10^\circ$ and $10^\circ - 15^\circ$ for 16/9 point means however appear to hold for satellite heights from 400 km to 1600 km.

Apart from the above strictly numerical point of view, we may also consider the optimum choice of spherical radius ψ^* for 16pt/9pt mean computation from the symmetry and the number of mean gravity anomaly blocks falling within the chosen value of ψ^*. In Figure 1, we have considered three cases for the location of the satellite subpoint in relation to the mean gravity anomaly blocks;

(1) At the common corner of 4 blocks,
(2) In the middle of the common edge of 2 blocks,
(3) In the center of a block.

Further, as the longitudinal extent of the blocks increase away from equator, the equatorial case above has been shown. At other locations of the satellite subpoint away from the equator, the number of mean gravity anomaly blocks with their centers (ψ_g, λ_g) within the specified ψ^* will only be less.

For each of the three cases, circles of radii $\psi^* = 5^\circ$ to 20° after every $2.^\circ5$, have been drawn, and the number of mean gravity anomaly blocks, whose centers fall within these are shown in Table 5.

TABLE 5

No. of Mean Gravity Anomaly Blocks for given ψ^*($5^\circ < \psi^* \leq 20^\circ$)

ψ^*	Case 1	Case 2	Case 3
7.5° - 10°	0	2	1
12.5°	4	2	1
17.5°	4	6	5
20°	4	6	5

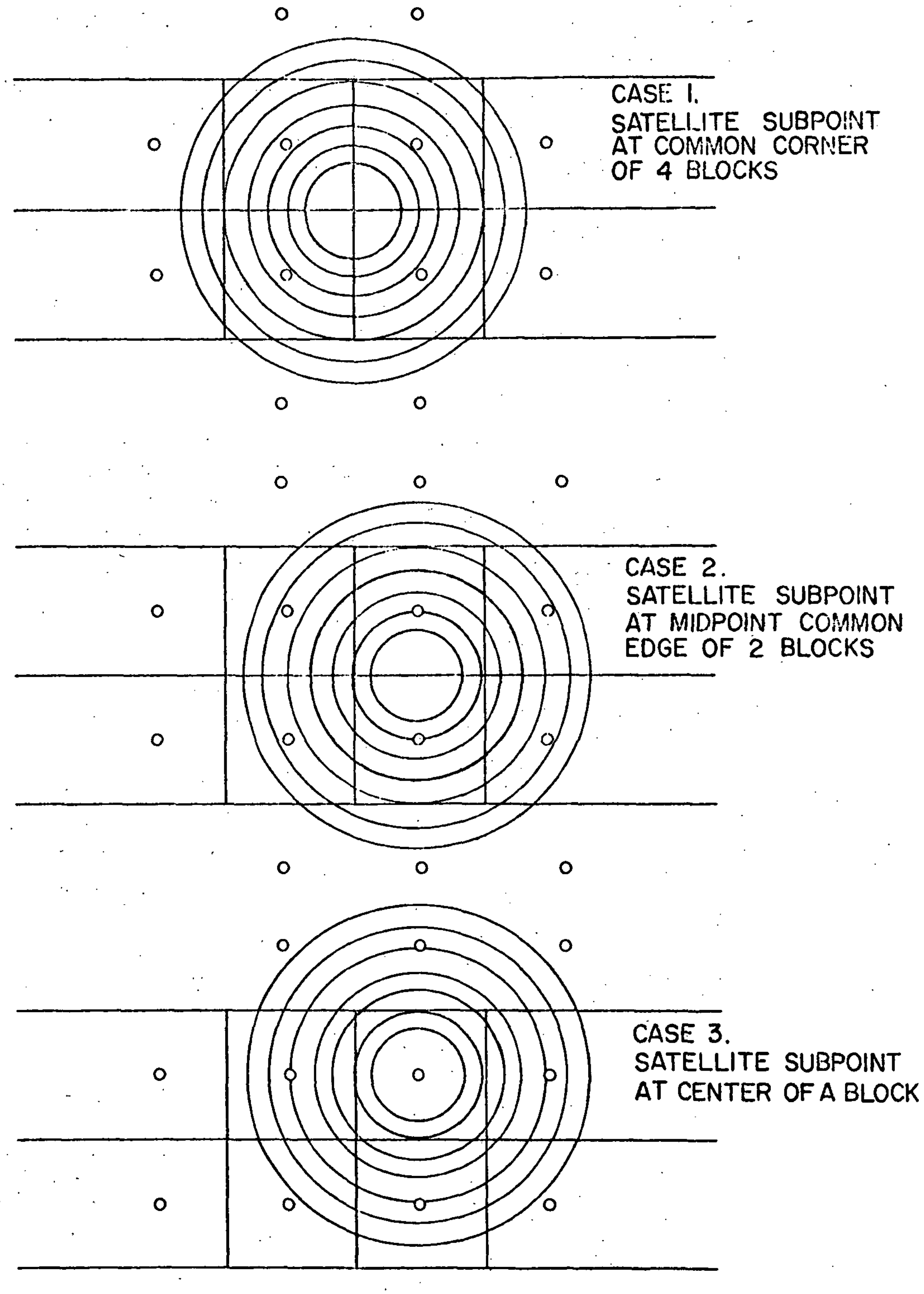


Figure 1 ψ^* From 5° To 20° From Satellite Subpoint

From this, and in view of the numerical results in Table 1, we may choose to have 16 point mean for $0^\circ < \psi^* < 10^\circ$ and 9 point mean $10^\circ \le \psi^* \le 20^\circ$, the slight increase in ψ^* not appreciably increasing the number of blocks.

The maximum number of mean gravity anomaly blocks requiring computation of 16 pt/9 pt mean would then be 1 or 2 and an additional 2 to 4 respectively, depending on the location of the satellite subpoint. When this happens to be within 5° of the pole, there will be only 3 blocks for 16/9 pt mean computation.

7. Number of Blocks for Computation of 4 Point Means

We now examine in Tables 6, 7 and 8 for satellite heights of approximately 400, 800 and 1600 km, the values of C_x, C_y, C_z for those mean gravity anomaly blocks, whose centers (φ_g, λ_g) are at spherical distance ψ^* of 20° - 45° from the satellite subpoint. For satellite height of about 400 km, 1 point value agrees with 16 point mean within about 2.5%, only after $\psi^* > 40^\circ$. For the same tolerance, $\psi^* > 35^\circ$ for satellite height of about 800 km, and $\psi^* > 30^\circ$ for satellite height of about 1600 km. We may therefore compute 4 point means for the mean gravity anomaly blocks whose centers are within the above spherical radius ψ^* from the satellite subpoint and compute only the center (1 point) value after that.

TABLE 6

C_x, C_y, C_z Values for $20^\circ < \psi^* < 45^\circ$ Satellite Ht. $\approx$ 400 km

Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	Difference 16P-1P	% (16P-1P)100 / 16P.	Difference 16P-4P	% (16P-4P)100 / 16P	Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	Difference 16P-1P	% (16P-1P)100 / 16P	Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	% (16P-1P)100 / 16P	Remarks
C_x Values																$r \simeq$ 6779 km
172	22.1	-.1044	-.0048	4.6	-.0010	1.0	156	33.2	-.0551	-.0004	0.6	95	38.4	.0377	0.1	$h \simeq$ 408 km
141	25.6	.0719	.0004	0.6	.0001	0.1	160	33.3	.0329	.0003	1.0	142	40.0	.0386	0.1	t = 1.0 min
138	25.9	-.0589	-.0010	1.7	-.0002	0.3	181	33.4	-.0357	-.0014	3.9	120	40.0	.0438	0.1	
173	27.8	-.0193	-.0010	5.4	-.0002	1.0	171	36.3	-.0412	-.0004	1.1	182	45.1	.0012	53.2*	*Large %
119	28.3	.0664	.0003	0.5	.0001	0.2	174	37.0	.0122	.0002	1.8	115	46.2	-.0124	1.4	value because of
93	33.0	.0026	-.0002	6.2	-.0001	3.8	137	37.2	-.0365	-.0002	0.6	96	46.9	.0326	0.2	
94	33.2	.0278	-.0001	0.3	.0000	0.0	116	37.9	-.0133	-.0002	1.5	155	47.4	-.0207	0.6	small value of 16pt. mean
C_y Values																
172	22.1	.0390	.0016	4.1	.0005	1.3	156	33.2	-.0174	.0002	1.4	95	38.4	-.0166	1.9	
141	25.6	.0547	.0011	2.0	.0002	0.4	160	33.3	.0377	.0006	1.6	142	40.0	.0129	2.2	
138	25.9	-.0434	.0002	0.4	.0001	0.2	181	33.4	.0227	.0013	5.8	120	40.0	.0017	18.7*	
173	27.8	.0602	.0022	3.7	.0005	0.8	171	36.3	-.0027	.0001	4.9	182	45.1	.0085-	17.6*	
119	28.3	.0126	.0007	5.4	.0002	1.6	174	37.0	.0297	.0013	4.4	115	46.2	-.0339	0.7	
93	33.0	-.0387	.0003	0.7	.0001	0.3	137	37.2	-.0321	.0003	0.8	96	46.9	-.0143	2.0	
94	33.2	-.0275	.0003	1.1	.0000	0.0	116	37.9	-.0391	.0003	0.7	155	47.4	-.0206	1.1	
C_z Values																
172	22.1	.0326	-.0012	3.8	-.0002	0.6	156	33.2	.0080	-.0005	6.6	95	38.4	-.0204	2.1	
141	25.6	-.0079	-.0010	13.2	-.0002	2.5	160	33.3	.0267	-.0006	2.2	142	40.0	.0157	2.4	
138	25.9	-.0525	-.0015	2.9	-.0003	0.6	181	33.4	.0424	.0000	0.1	120	40.0	.0012	30.3*	
173	27.8	.0516	-.0001	0.1	.0000	0.0	171	36.3	.0267	-.0005	1.8	182	45.1	.0386	0.2	
119	28.3	-.0372	-.0010	2.7	-.0002	0.5	174	37.0	.0389	-.0002	0.6	115	46.2	-.0002	160.6*	
93	33.0	-.0439	-.0007	1.5	-.0002	0.4	137	37.2	-.0048	-.0004	8.5	96	46.9	.0010	28.8*	
94	33.2	-.0428	-.0007	1.5	-.0001	0.2	116	37.9	-.0021	-.0005	2.0	155	47.4	.0203	1.5	

TABLE 7

C_x, C_y, C_z Values for $20^\circ < \psi^* < 45^\circ$ Satellite Ht. $\approx$ 800 km

Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	Difference 16P-1P	% (16P-1P)100 / 16P.	Difference 16P-4P	% (16P-4P)100 / 16P	Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	Difference 16P-1P	% (16P-1P)100 / 16P	Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	% (16P-1P)100 / 16P	Remarks
C_x Values																
172	22.1	-.0838	-.0028	3.3	-.0005	0.6	156	33.1	-.0470	-.0002	0.5	95	38.3	.0307	0.3	r ≃ 7187km
138	25.6	-.0507	-.0006	1.2	-.0001	0.2	181	33.6	-.0299	-.0011	3.6	120	40.2	.0355	0.3	h ≃ 816km
141	25.9	.0548	-.0002	0.4	-.0001	0.2	160	33.6	.0258	.0002	0.6	142	40.2	.0312	0.3	t = 1.0 min.
173	28.1	-.0165	-.0003	5.1	-.0001	0.6	171	36.3	-.0350	-.0003	1.0	182	45.4	.0005	105.2*	*Large %
119	28.4	.0522	-.0001	0.3	.0000	0.1	137	37.0	-.0320	-.0002	0.5	115	45.9	-.0115	1.2	value be-
93	32.7	.0010	-.0002	19.0	.0000	2.9	174	37.4	.0093	.0001	1.3	96	47.0	.0268	0.3	cause of
94	33.0	.0221	-.0002	0.8	.0000	0.2	116	37.6	-.0124	-.0002	1.4	155	47.2	-.0184	0.5	small value
																of 16pt. mean
C_y Values																
172	22.1	.0391	.0019	4.7	.0005	1.3	156	33.1	-.0112	.0003	3.0	95	38.3	-.0116	2.9	
138	25.6	-.0293	.0009	3.0	.0002	0.7	181	33.6	.0212	.0012	5.5	120	40.2	.0031	9.3*	
141	25.9	.0480	.0009	1.9	.0002	0.4	160	33.6	.0325	.0005	1.5	142	40.2	.0121	2.1	
173	28.1	.0518	.0018	3.5	.0004	0.8	171	36.3	.0002	.0002	100.4*	182	45.4	.0083	15.4*	
119	28.4	.0144	.0007	5.1	.0001	0.7	137	37.0	-.0243	.0003	1.3	115	45.9	-.0271	0.9	
93	32.7	-.0289	.0004	1.5	.0001	0.3	174	37.4	.0259	.0011	4.2	96	47.0	-.0105	2.5	
94	33.0	-.0196	.0004	2.2	.0001	0.5	116	37.6	-.0304	.0003	1.0	155	47.2	-.0158	1.3	
C_z Values																
172	22.1	.0192	-.0022	11.5	-.0005	2.6	156	33.1	.0041	-.0005	13.0	95	38.3	-.0188	2.0	
138	25.6	-.0480	-.0013	2.6	-.0002	0.4	181	33.6	.0325	-.0003	0.9	120	40.2	-.0004	85.7*	
141	25.9	-.0098	-.0011	11.2	-.0003	3.1	160	33.6	.0199	-.0006	3.0	142	40.2	.0117	3.0	
173	28.1	.0379	-.0006	1.5	-.0001	0.3	171	36.3	.0203	-.0005	2.4	182	45.4	.0308	0.1	
119	28.4	-.0330	-.0009	2.6	-.0001	0.3	137	37.0	-.0064	-.0004	6.0	115	45.9	-.0021	12.7*	
93	32.7	-.0396	-.0005	1.4	-.0001	0.3	174	37.4	.0303	-.0003	1.1	96	47.0	-.0004	57.4*	
94	33.0	-.0383	-.0005	1.4	-.0002	0.5	116	37.6	-.0211	-.0004	1.9	155	47.2	.0158	1.7	

TABLE 8

C_x, C_y, C_z Values for $20^\circ < \psi^* < 45^\circ$ Satellite Ht. $\approx$ 1600 km

Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	Difference 16P-1P	% (16P-1P)100 / 16P.	Difference 16P-4P	% (16P-4P)100 / 16P	Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	Difference 16P-1P	% (16P-1P)100 / 16P	Block No.	Sph. Dist. ψ^* Deg.	16 Pt. Mean	% (16P-1P)100 / 16P	Remarks
C_x Values																r ≃ 7965km
172	22.1	-.0522	-.0001	0.3	.0000	0.1	156	32.8	-.0339	.0001	0.2	95	38.2	.0207	0.9	h ≃ 1594km
138	25.1	-.0356	.0002	0.5	.0001	0.2	181	33.8	-.0211	-.0006	2.7	120	40.5	.0241	0.8	t = 1.0 min
141	26.3	.0322	-.0007	2.2	-.0002	0.5	160	34.1	.0163	.0000	0.2	142	40.7	.0209	0.7	
173	28.5	-.0118	-.0005	3.9	-.0001	0.7	171	36.2	-.0253	-.0001	0.5	115	45.4	-.0096	0.8	
119	28.5	.0324	-.0006	1.8	-.0001	0.4	137	36.5	-.0241	.0000	0.0					
93	32.2	-.0007	-.0002	23.5*	.0000	5.5*	116	37.0	-.0101	-.0001	0.9					
94	32.7	.0141	-.0003	1.9	-.0001	0.4	174	37.9	.0056	.0000	0.3					
C_y Values																
172	22.1	.0320	.0009	2.7	.0002	0.6	156	32.8	-.0039	.0003	9.0	95	38.2	-.0056	5.4*	* Large %
138	25.1	-.0119	.0012	10.0	.0002	2.1	181	33.8	.0174	.0008	4.4	120	40.5	.0042	5.0*	value be-
141	26.3	.0352	.0002	0.7	.0001	0.1	160	34.1	.0241	.0002	1.0	142	40.7	.0101	1.6	cause of
173	28.5	.0374	.0010	2.6	.0002	0.5	171	36.2	.0029	.0002	5.6*	115	45.4	-.0177	1.3	small value
119	28.5	.0142	.0005	3.4	.0001	0.7	137	36.5	-.0141	.0003	2.5					of 16pt.mean
93	32.2	-.0160	.0005	3.3	.0001	0.6	116	37.0	-.0186	.0004	2.0					
94	32.7	-.0097	.0005	4.8	.0001	1.0	174	37.9	.0195	.0007	3.4					
C_z Values																
172	22.1	.0051	-.0020	39.4	-.0004	8.0*	156	32.8	.0000	-.0004	29566.5*	95	38.2	-.0152	1.4	
138	25.1	-.0368	-.0004	1.1	-.0001	0.1	181	33.8	.0196	-.0005	2.7	120	40.5	-.0018	13.2*	
141	26.3	-.0099	-.0008	7.7	-.0002	1.7	160	34.1	.0114	-.0005	4.7	142	40.7	.0067	4.2*	
173	28.5	.0208	-.0009	4.3	-.0002	0.9	171	36.2	.0121	-.0004	3.7	115	45.4	-.0037	5.0*	
119	28.5	-.0248	-.0004	1.6	-.0001	0.3	137	36.5	-.0072	-.0003	3.8					
93	32.2	-.0309	-.0002	0.7	-.0001	0.2	116	37.0	-.0178	-.0002	1.2					
94	32.7	-.0294	-.0002	0.7	.0000	0.1	174	37.9	.0190	-.0004	2.2					

We may now again consider the optimum choice of ψ^* from the symmetry and number of blocks falling within a specified value of ψ^*. Figures 2, 3, 4 show circles with radii $\psi^* = 30°, 35°, 40°, 45°$ for the cases 1, 2, 3 of the satellite subpoint location, as discussed in Section 6. These figures are for the satellite subpoint being on the equator. Figure 5 shows the location of mean anomaly gravity blocks around either pole. The number of blocks when the satellite subpoint is in mid-latitudes will be somewhere between the equatorial and polar cases. The results for the latter two cases have been summarized in Table 9 below.

TABLE 9

Number of Mean Gravity Anomaly Blocks for given ψ^* $(30° \le \psi^* \le 45°)$

ψ^*	Fig. 2	Fig. 3	Fig. 4	Fig. 5
30°	12	12	13	12
35°	16	16	18	12
40°	24	22	21	27
45°	24	26	24	27

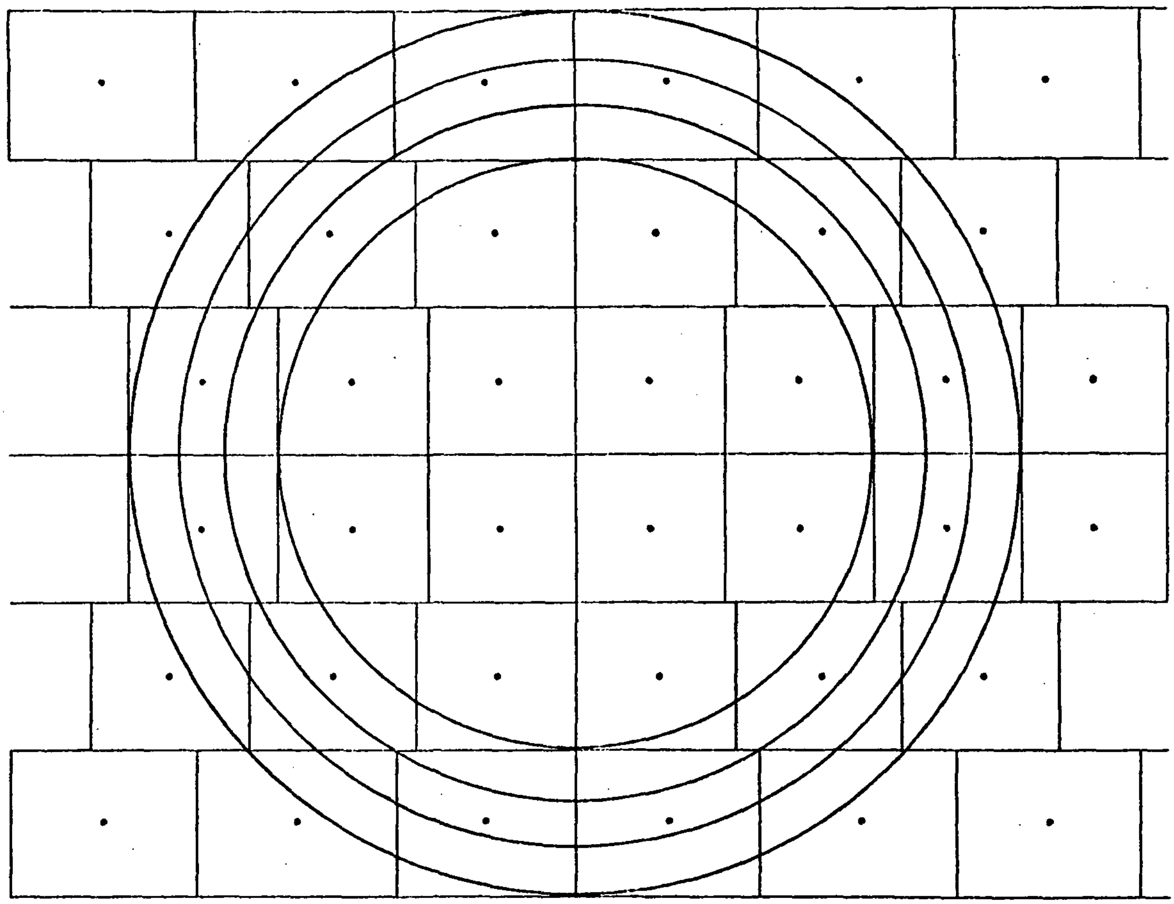


Figure 2 $30^{\circ} \leq \psi^{*} \leq 45^{\circ}$ Equatorial Location Case 1

Satellite Subpoint at Common Corner of 4 Blocks

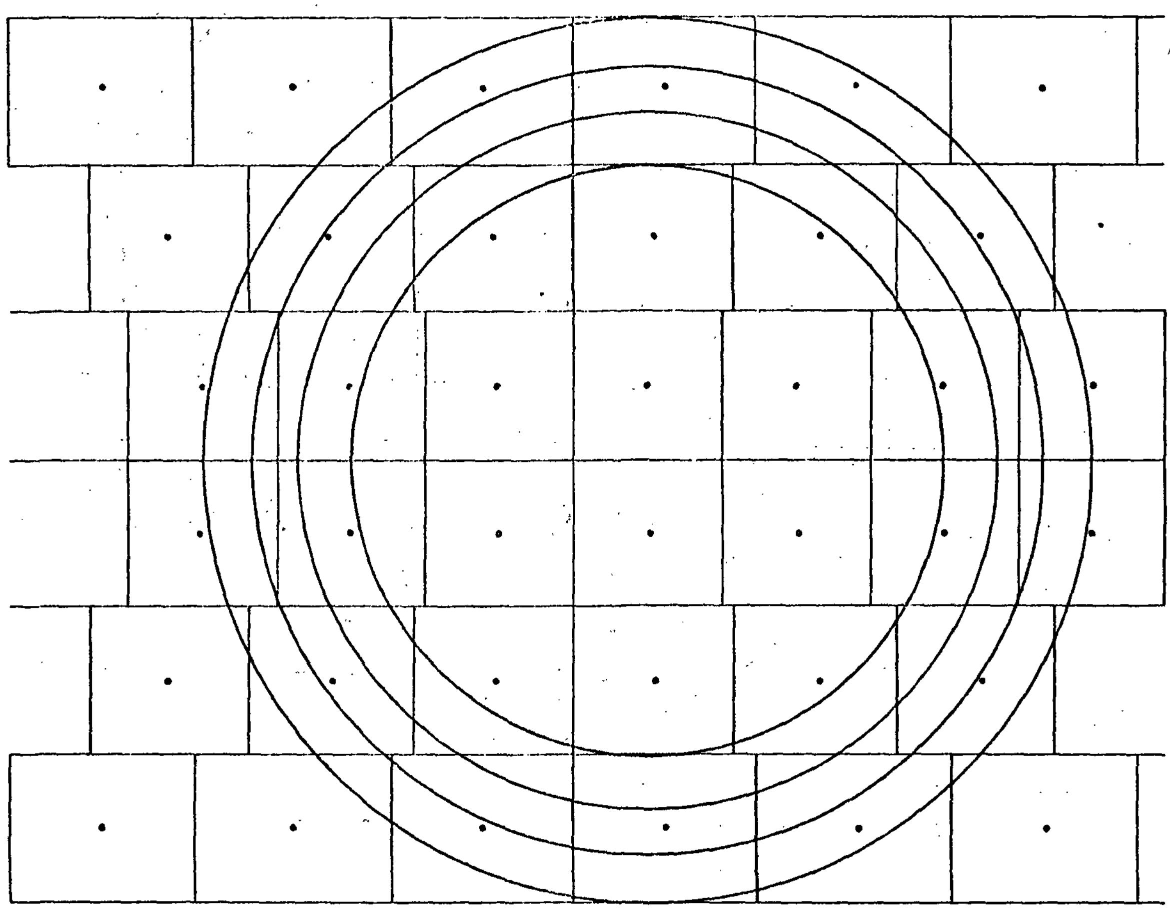


Figure 3 $30^\circ \leq \psi* \leq 45^\circ$ Equatorial Location Case 2

Satellite Subpoint at Midpoint of Common Edge of 2 Blocks

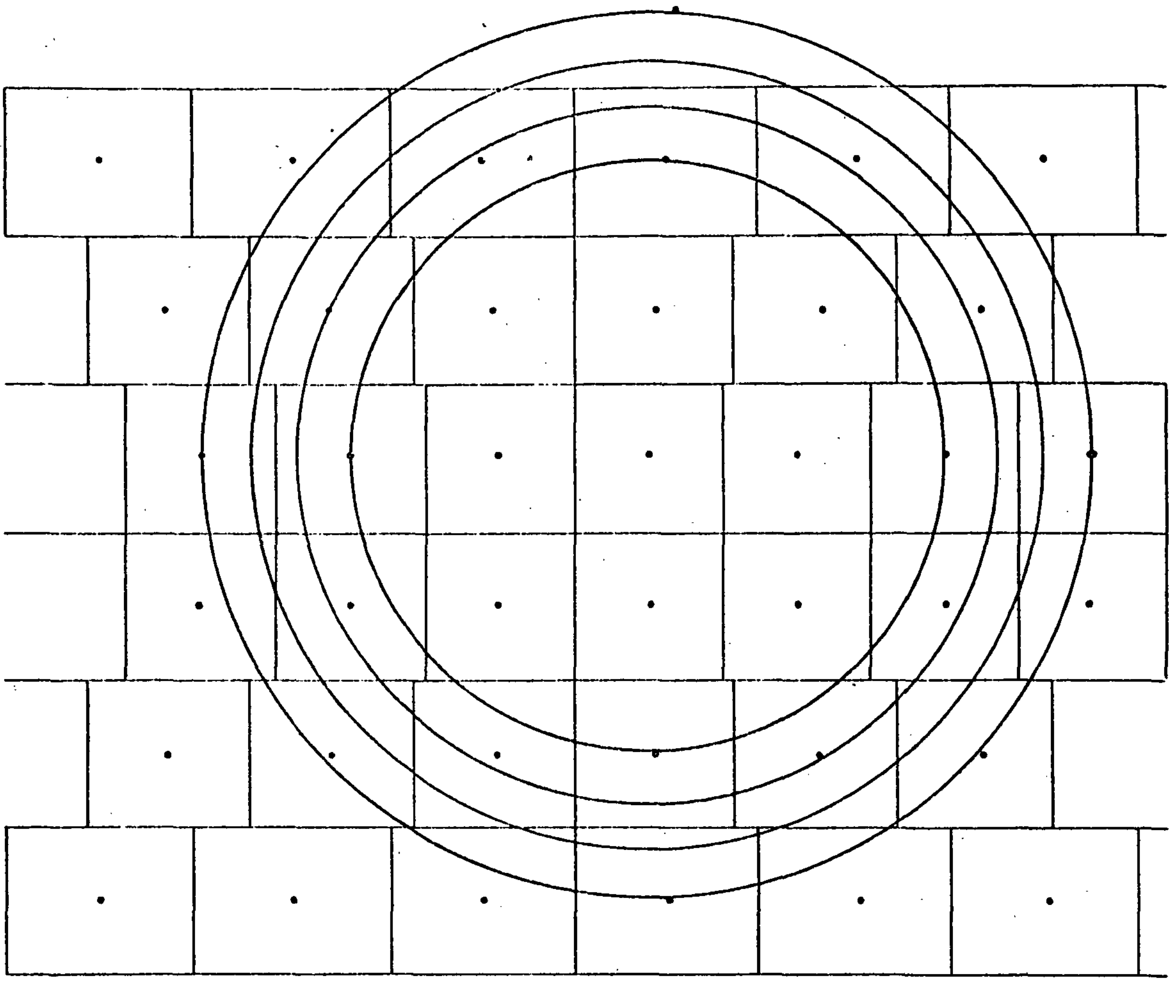


Figure 4 $30^\circ \leq \psi^* \leq 45^\circ$ Equatorial Location Case 3

Satellite Subpoint at the Center of a Block

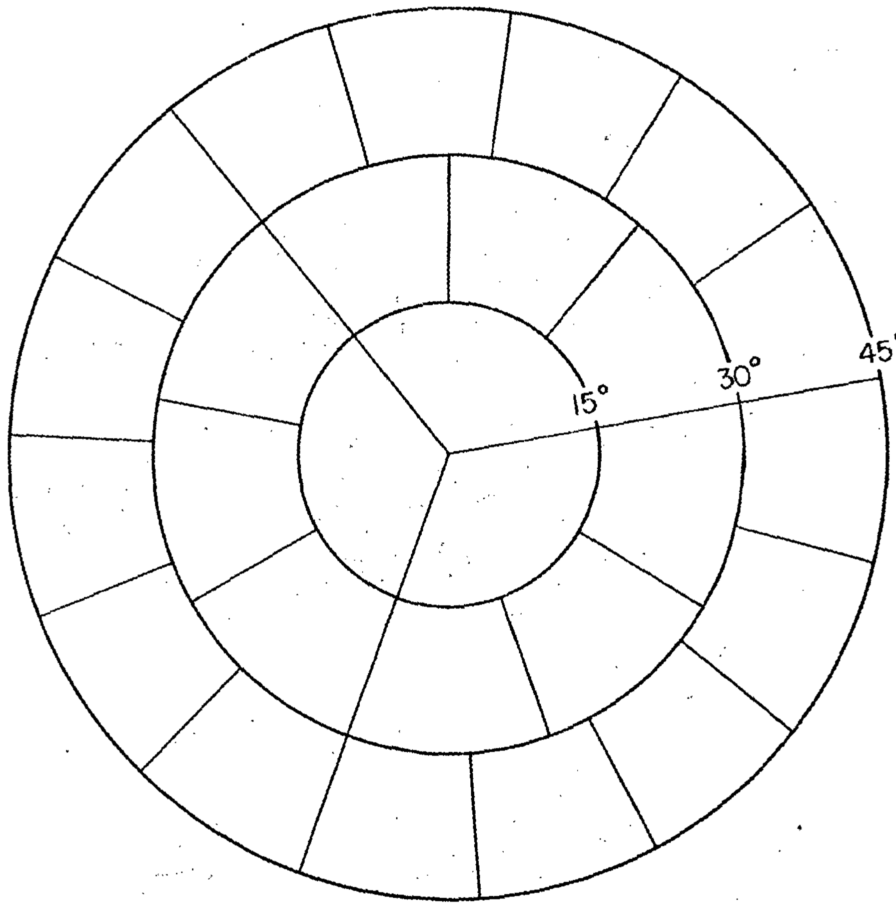


Figure 5 $15^\circ \times 15^\circ$ Equal Area Mean Gravity Anomaly Blocks Around Pole

$30^\circ \leq \psi^* \leq 45^\circ$

Clearly, for satellites, with heights above 1600 km or so, it will be adequate to take 4 point means for $\psi^* \leq 30^\circ$, the number of blocks being 12 to 13. This number includes the 4 to 6 blocks for which 9/16 point means will be taken as discussed in Section 6. For satellites of height of about 800 km, we may take 4 point means for $\psi^* \leq 35^\circ$, the number of blocks being 12 to 18, usually 16. However, for satellites of still lower heights, the number of blocks for $\psi^* \leq 40^\circ$ is 21 to 27, which is not significantly different from the number of blocks for $\psi^* \leq 45^\circ$, being 24 to 27.

It is therefore worthwhile to compute 4 point means for all blocks, whose centers are within a spherical radius $20^\circ < \psi^* \leq 45^\circ$ from the satellite subpoint, if the height of the satellite is lower than 800 km. For satellite heights 800 km to 1600 km, $20^\circ < \psi^* \leq 35^\circ$. For higher satellites, 4 point means may be taken for $20^\circ < \psi^* \leq 30^\circ$.

8. Computation of Blocks at $\psi^* > 135^\circ$

We now examine if the values of C_x, C_y, C_z for the mean gravity anomaly blocks, which are far away from the satellite subpoint, say at spherical distance $\psi^* > 135^\circ$, are so small as compared to the average values, that they may be ignored, i.e. not computed, and assumed to be zero.

The root mean square value of C_x, C_y, C_z was accordingly first examined for all the 184 blocks, and then for the remaining blocks after respectively neglecting the blocks with $\psi^* > 165^\circ$, $> 150^\circ$, $> 135^\circ$. The results for satellite height of about 800 km are given in Table 10 below.

TABLE 10

Variation of RMS Value of C_x, C_y, C_z as Blocks with $\psi > 135^\circ$ are Neglected
Satellite Height $\approx$ 800 km

Root Mean Square Values				Percentage Variation in RMS Values			Remarks
ψ^*up to 180°	ψ^*up to 165°	ψ^*up to 150°	ψ^*up to 135°	ψ^*up to 165°	ψ^*up to 150°	ψ^*up to 135°	
C_x Values							$r \simeq 7187$ km
.0251	.0253	.0259	.0270	0.7	3.2	7.6	$h \simeq 816$ km
No. of Blocks Neglected				3	13	27	t = 1.0 min
C_y Values							
.0293	.0294	.0300	.0310	0.4	2.4	5.9	
No. of Blocks Neglected				3	13	27	
C_z Values							
.0302	.0303	.0310	.0321	0.6	2.7	6.5	
No. of Blocks Neglected				3	13	27	

We therefore find that the values of C_x, C_y, C_z are not negligibly small for large values of ψ^*, when compared with their average values. This is seen more explicitly by listing the values of C_x, C_y, C_z in Table 11 for the blocks at spherical distance ψ^* exceeding 135°. We find that the values do not decrease uniformly with increase in ψ^*, and there are several values, which are more than half of the root mean square value.

Figure 6 displays these results graphically, and in particular, shows that the values are in fact larger than for other blocks, for example, for blocks with ψ^* from 75° to 90°, whose values have also been shown in the same graph. The results for other satellite heights are similar and have thus not been shown.

We thus cannot dispense with the computation of C_x, C_y, C_z values for large values of ψ^*, though it is adequate to compute them only for the center (1 point value) of the block, as discussed in Section 7.

TABLE 11

C_x, C_y, C_z Values for $\psi^* > 135^{\circ}$
Satellite Ht. $\approx$ 800 km

Block No.	Sph. Dist. ψ*Deg.	C_x	C_y	C_z	Block No.	Sph. Dist. ψ*Deg	C_x	C_y	C_z	Remarks
10	135.2	-.0004	.0195	.0013	19	150.9	-.0154	.0155	-.0085	r ≃ 7187 km
18	136.4	-.0160	.0101	-.0019	8	151.0	-.0113	.0211	-.0061	h ≃ 816 km
2	136.9	-.0088	.0187	.0016	60	151.6	.0031	.0162	-.0180	t = 1.0 min
23	139.1	.0031	.0193	-.0032	22	153.6	-.0015	.0222	-.0094	
7	139.6	-.0139	.0157	-.0009	36	154.1	-.0161	.0131	-.0140	
35	139.8	-.0175	.0079	-.0075	57	154.4	-.0128	.0101	-.0192	
61	139.8	.0081	.0146	-.0122	59	160.7	-.0026	.0158	-.0214	
83	141.7	.0050	.0091	-.0183	39	161.4	-.0011	.0211	-.0164	
80	142.4	-.0096	.0031	-.0187	58	162.1	-.0082	.0137	-.0218	
56	143.0	-.0158	.0056	-.0140	20	163.9	-.0121	.0120	-.0125	
40	146.9	.0042	.0196	-.0109	21	165.5	-.0070	.0224	-.0129	
82	147.0	-.0003	.0084	-.0211	37	168.7	-.0123	.0175	-.0180	
81	147.3	-.0054	.0063	-.0212	38	175.7	-.0070	.0203	-.0189	
9	148.7	-.0056	.0227	-.0051		RMS Value	.0251	.0293	.0302	

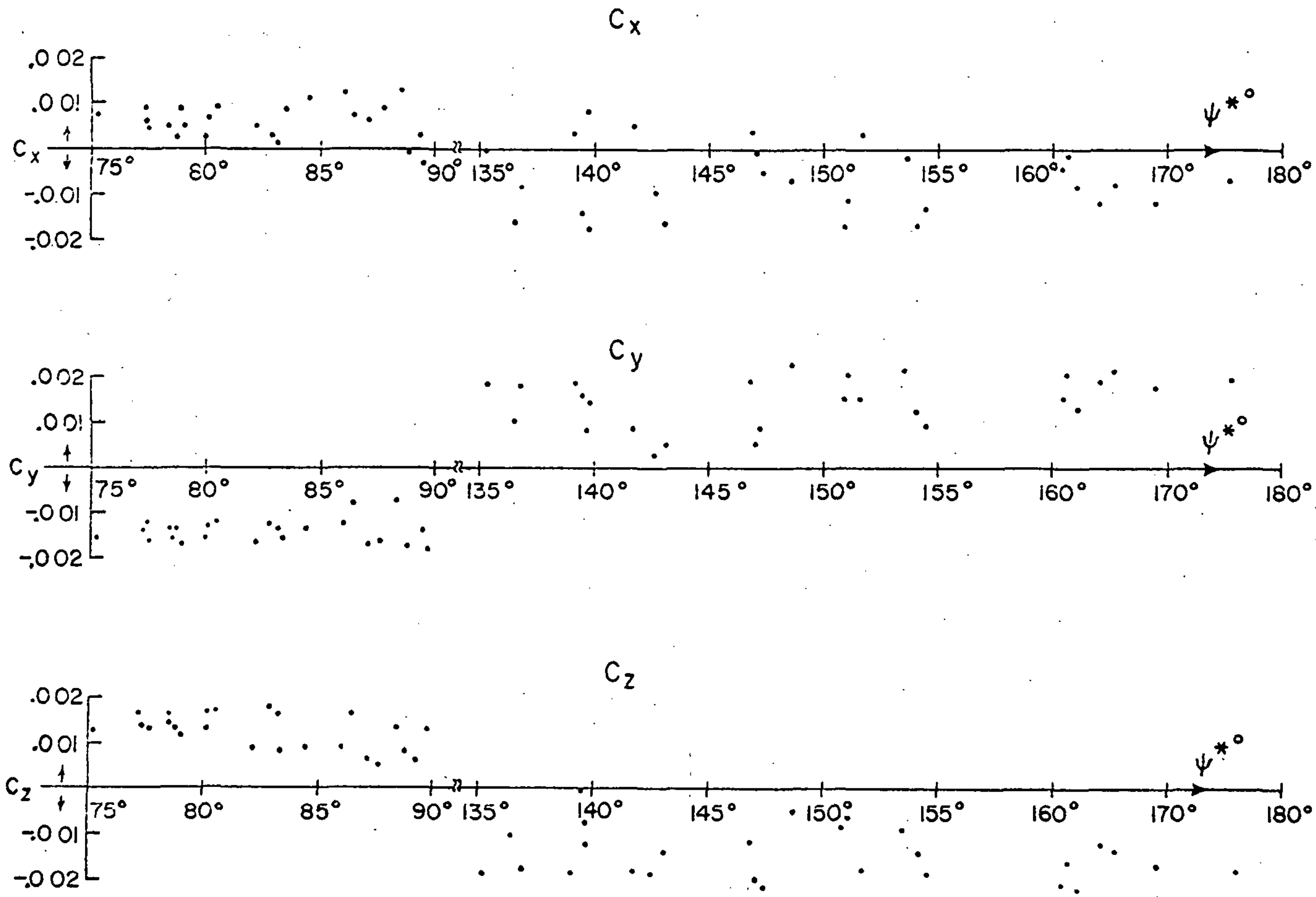


Figure 6 C_x, C_y, C_z Values

Satellite Ht. ≈ 800 km

9. Timing of Computer Runs

Computer runs were made on IBM System/370 Model 165 computer for the computation of C_x, C_y, C_z for a specified position of the satellite for all the 184 $15^\circ \times 15^\circ$ mean gravity anomaly blocks as per equations (9) to (12) substituted in equation (7). The actual CPU time was noted for the following cases:

(a) Center point value of C_x, C_y, C_z for all 184 blocks

(b) Four point mean value of C_x, C_y, C_z for all 184 blocks

(c) Nine point mean value of C_x, C_y, C_z for all 184 blocks

(d) Sixteen point mean value of C_x, C_y, C_z for all 184 blocks.

The CPU execution time was actually obtained for 10 loops each of (a) to (d) above, and after dividing by 10, the time was respectively 0.034, 0.155, 0.334 and 0.597 seconds. These times do not include the time for the preliminary/subsequent computation, or for any write statements etc., which are the same for any of the cases (a) to (d). The difference in time is solely due to the computation being done for 1, 4, 9 or 16 points in the blocks, and obtaining a mean value.

The CPU execution time was then similarly obtained for computing C_x, C_y, C_z values according to the scheme recommended in Section 6, 7 and 8. This, for a specified satellite position, required computation of C_x, C_y, C_z as 16 point mean for 2 blocks, 9 point mean for 3 blocks, 4 point mean for 11 blocks and center point value for the remaining 168 blocks, and the time was 0.055 seconds.

It was generally felt, before this investigation was taken up, that the computation of C_x, C_y, C_z would not be of adequate accuracy if done only at the center point of a $15^\circ \times 15^\circ$ mean gravity anomaly block. The usual safeguard would then be to take a 4 point mean for all the 184 blocks. As we realize now, this would still give inaccurate results for the nearest few blocks, and would take $0.155 - 0.055 = 0.1$ seconds additional time for each satellite position. If we needed the computation at intervals of one minute for some orbital analysis studies, the extra CPU time for each 24 hour simulated orbit would be about

2.5 minutes.

Let us consider the extreme case of the requirement as per Tables 5 and 9, of computation of 16 point mean for 2 blocks, 9 point mean for 4 blocks, 4 point mean for 21 blocks and center point value for 157 blocks. The saving in CPU execution time as compared to 4 point mean for all blocks would then still be 0.09 seconds for each satellite position. At the same time, it will ensure that no errors larger than about 2.5% are being caused for any block, which is not the case if 4 point mean is being taken for all blocks.

10. Summary and Conclusions

The computation of partial derivatives of the disturbing force of the earth's gravity field with respect to individual gravity anomalies is required to be computed for several orbital and trajectory analysis studies of artifical earth satellites used for geodetic purposes. We have here considered the case of solution of equation of motion of a satellite only affected by the earth's gravitational force, which is described in terms of $15^\circ \times 15^\circ$ equal area mean gravity anomaly blocks, as referred to a defined reference surface. For convenience, we define the partial derivatives of the components λ, in an inertial coordinate system λ, of the disturbing force C_x, C_y, C_z given by:

$$C_{x_i} = \frac{\partial}{\partial \Delta g_i}\left(\frac{\partial T}{\partial X}\right), \quad C_{y_i} = \frac{\partial}{\partial \Delta g_i}\left(\frac{\partial T}{\partial Y}\right), \quad C_{z_i} = \frac{\partial}{\partial \Delta g_i}\left(\frac{\partial T}{\partial Z}\right)$$

where T is the disturbing potential of the earth's gravity field and Δg_i is an individual mean gravity anomaly over one of the 184 $15^\circ \times 15^\circ$ equal area blocks. (For details see Section 3).

We have examined in this report the numerical evaluation of C_x, C_y, C_z. The principle criterion has been the spherical distance ψ^* of the center of the block from the satellite subpoint. It has been found that for the blocks nearest to the satellite subpoint, the value of C_x, C_y, C_z are in considerable error, even more than 50% for $\psi^* < 10^\circ$ (Tables 2 to 4; λ, if computed only for the center of the block. This error reduces to about 2.5% for ψ^* exceeding $30^\circ - 40^\circ$, depending on the height of the satellite (Tables 6 to 8), as compared to 16 point mean, described below.

For these blocks near the satellite subpoint, the value of C_x, C_y, C_z can be computed within a tolerance of 1 to 3%, if the value is computed at several points in the block and then meaned. It has been found that these subdivisions of blocks should by symmetric with respect to latitude and longitude, thus giving 4, 9 or 16 subdivisions and thus 4 point, 9 point and 16 point means for consideration. For $\psi^* < 5^\circ$, 9 point mean is in error by 1 to 4% as compared to the 16 point mean. It therefore appears advisable to take 16 point mean for a few of the nearest blocks.

Four point means compare with 16 point means within about 2% for $\psi^* > 15^\circ$ to 20°, and for blocks nearer to the satellite subpoint, 9 point means require to be taken. The actual limiting value of ψ^* for taking 16/9 point means may be chosen, in view of above figures, after considering the number of blocks which are likely to occur for ψ^* from 5° to 20° at various latitudes. From Figure 1 and Table 5, the limiting values are found to be 10° for 16 point mean and 20° for 9 point mean. The maximum number of blocks for $\psi^* < 10^\circ$ is two, which would have increased to four, if this limit was exceeded. Similary, the maximum number of blocks for $\psi^* < 20^\circ$ is six (inclusive of blocks for 16 point mean) which would have increased to nine if this limiting value of ψ^* was to exceed 20°.

The computation of 16 point mean for $\psi^* < 10^\circ$ and 9 point mean for $10^\circ \le \psi^* \le 20^\circ$ appears to hold for satellite heights from 400 km to 1600 km above the earth. For higher satellites, 16 point means may be dispensed with and 9 point means may be taken for $\psi^* < 20^\circ$.

The limiting values of ψ^* for taking 4 point means shows greater variation with the height of satellite, as discussed in Section 7. This has been found to be $20^\circ < \psi^* \le 30^\circ$ for satellite height exceeding 1600 km, $20^\circ < \psi^* \le 35^\circ$ for height from 800 - 1600 km, and $20^\circ < \psi^* \le 45^\circ$ for lower satellites. The total number of blocks corresponding to these limits would be 12 to 13, 12 to 18, 24 to 27 respectively. These numbers include 4 to 6 blocks for which 9/16 point means would be taken.

The values of C_x, C_y, C_z do not decrease uniformly with increase in ψ^*, and, in particular, it is not possible to dispense with the computation of values for those mean gravity anomaly blocks, which are near the antipode, the point at a spherical distance of $\psi^* = 180^\circ$ from the satellite subpoint. (For details see Section 9).

Finally, the actual CPU execution time was checked on IBM System/370 Model 165 Computer for the 16/9/4 point mean and center point computation of C_x, C_y, C_z, as per limiting values of ψ^* discussed above. It was found to have taken 0.1 seconds less for a single satellite position, as compared to the 4 point mean for all the blocks. The saving of time, for example, in the generation of satellite orbits over extended periods by numerical integration approach, would be noticeable. Further, the 4 point mean would have caused errors for the nearest blocks of 2 to 20%, while the errors in the scheme suggested now would be less than about 2.5% for all blocks.

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